Phase transitions and gravitational waves

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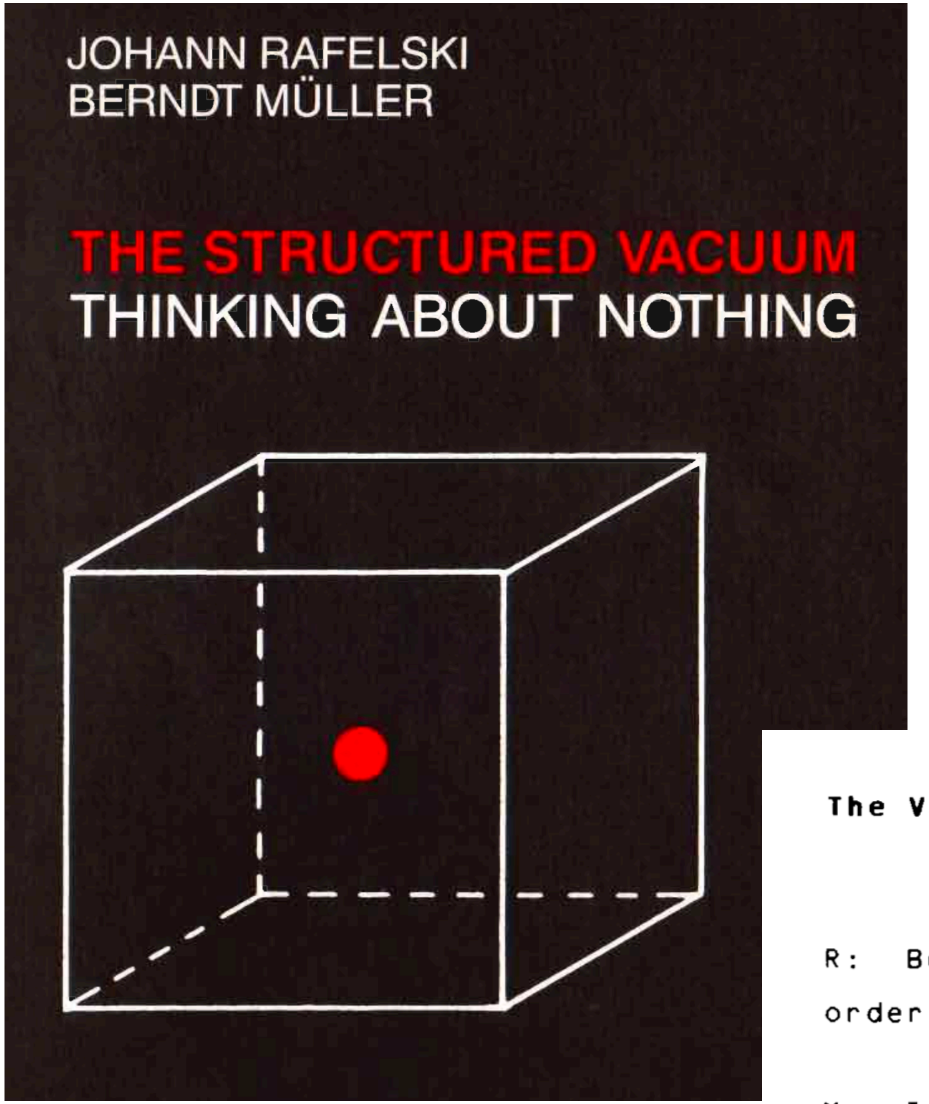
Dark Matter, Phase transitions and Gravitational Waves

The zero temperature of the zero order potential



The Vacuum and the Laws of Nature

R: But is understanding of the vacuum important in order to understand the laws of physics?

M: Indeed so. The surprising thing that we have learnt in the last decade is that the vacuum is very important in understanding the laws of physics and that it comes in addition to the laws of physics. You may have the same set of laws of physics operating in a different vacuum and they would describe very different phenomena.

The SM potential and its minima

What you see in books

$$< \Phi_{SM} > = \binom{0}{v} \qquad Q_{SM} < \Phi_{SM} > = I_3 + \frac{Y}{2} < \Phi_{SM} > = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \binom{0}{v} = 0$$

What you don't see in books

$$< \Phi_{SM} > = \begin{pmatrix} v_1 + i v_2 \\ v_3 + i v_4 \end{pmatrix}$$

Now use the kinetic scalar term
to find the mass matrix of the gauge boson.

and you find the mass spectrum (for the gauge bosons)

$$m_1^2 = m_2^2 = \frac{g^2 v^2}{4} \qquad v^2 = v_1^2 + v_2^2 + v_3^2 + v_4^2$$

$$m_3^2 = \, = \frac{v^2}{4}(g^2 + g'^2 Y^2)$$

$$m_4^2 = 0$$

It's the photon!

So U(1) survives and charge is always conserved.
Is this obvious?

The SM potential and its minima

It is obvious because you can use the SU(2) freedom to perform the rotation

$$< \Phi_{SM} > = \begin{pmatrix} v_1 + i v_2 \\ v_3 + i v_4 \end{pmatrix} \quad \rightarrow \quad < \Phi_{SM} > = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Using a more general vacuum would just mean to redefine the charge operator.

For the same reason any phase in the vacuum can be rotated away. This means that no spontaneous CP can occur. And the potential is also explicitly CP conserving.

The SM has no CB and no CP violation in the potential.

The result also holds for any extension with singlets with Y=0 because they do not contribute to the mass matrix of the gauge bosons (CP case later).

Explicit breaking - if the Lagrangian is not invariant under a given symmetry

Spontaneous breaking - if the Lagrangian is invariant under a given symmetry but the vacuum is not

The 2HDM potential and its minima

Let us now extend the SM by adding a new complex doublet. The most general potential for the 2HDM invariant under $\Phi_1 \to \Phi_1; \quad \Phi_2 \to -\Phi_2$ and softly broken by the m^2_{12} term is

$$V = m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + h.c.)$$
$$+ \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2}\left[(\Phi_1^\dagger \Phi_2) + h.c.\right]$$

explicitly CP-conserving because m^2_{12} and λ_5 are real.

The most general vacuum structure is

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \; ; \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{cb} \\ v_2 + i v_{cp} \end{pmatrix}$$

- **CP conserving (N)** $\quad \langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \; ; \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$
- **Charge breaking (CB)** $\quad \langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1' \end{pmatrix} \; ; \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha \\ v_2' \end{pmatrix}$
- **CP breaking (CP)** $\quad \langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1' + i\delta \end{pmatrix} \; ; \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2' \end{pmatrix}$

Mass eigenstates - gauge bosons

- Gauge bosons

$$D_\mu \Phi = \partial_\mu \Phi - \frac{i}{2}\begin{bmatrix} gW^3_\mu + g'B_\mu & \sqrt{2}gW^+_\mu \\ \sqrt{2}gW^-_\mu & -gW^3_\mu + g'B_\mu \end{bmatrix}\Phi \qquad W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$$

$$< \Phi_1 > = \begin{pmatrix} v_a \\ v_b \end{pmatrix} \qquad < \Phi_2 > = \begin{pmatrix} 0 \\ v_c e^{i\theta} \end{pmatrix}$$

$$D_\mu \equiv \partial_\mu - i\frac{g'}{2}YB_\mu - i\frac{g}{2}\sigma_j W^j_\mu - i\frac{g_3}{2}\lambda_j G^j_\mu$$

Find the mass matrix in this model

$$\begin{cases} Z_\mu = c_W W^3_\mu - s_W B_\mu \\ A_\mu = s_W W^3_\mu + c_W B_\mu \end{cases} \qquad c_W = \frac{g}{\sqrt{g^2 + g'^2}} = \frac{M_W}{M_Z}$$

The 2HDM potential and its minima

Let us understand why. Now we have 2 doublets and 8 possible VEVs

$$< \Phi_k > = \begin{pmatrix} v_1^k + i v_2^k \\ v_3^k + i v_4^k \end{pmatrix}$$

We can use the SU(2) × U(1) freedom to write the most general form for the vacuum

$$< \Phi_1 > = \begin{pmatrix} v_a \\ v_b \end{pmatrix} \qquad < \Phi_2 > = \begin{pmatrix} 0 \\ v_c e^{i\theta} \end{pmatrix}$$

and you find the mass spectrum (for the gauge bosons)

$$m_1^2 = m_2^2 = \frac{g^2 v^2}{4} \qquad v^2 = v_a^2 + v_b^2 + v_c^2$$

$$m_3^2 = \; = \frac{1}{8}\left[v^2(g^2 + g'^2 Y^2) + \sqrt{v^4(g^2 + g'^2 Y^2)^2 - 16 g^2 g'^2 v_a^2 v_c^2 Y^2} \right]$$

$$m_4^2 = \frac{1}{8}\left[v^2(g^2 + g'^2 Y^2) - \sqrt{v^4(g^2 + g'^2 Y^2)^2 - 16 g^2 g'^2 v_a^2 v_c^2 Y^2} \right]$$

Is it the photon?

The 2HDM potential and its minima

Let us have a closer look at the photon mass

$$m_4^2 = \frac{1}{8}\left[v^2(g^2 + g'^2 Y^2) - \sqrt{v^4(g^2 + g'^2 Y^2)^2 - 16 g^2 g'^2 v_a^2 v_c^2 Y^2}\right]$$

There are two ways to recover a zero mass for the photon

$v_c = 0 \quad \Rightarrow \quad <\Phi_1> = \begin{pmatrix} v_a \\ v_b \end{pmatrix} \qquad <\Phi_2> = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ⇨ SM

$v_a = 0 \quad \Rightarrow \quad <\Phi_1> = \begin{pmatrix} 0 \\ v_b \end{pmatrix} \qquad <\Phi_2> = \begin{pmatrix} 0 \\ v_c e^{i\theta} \end{pmatrix}$ ⇨ Vacua are aligned

OR ELSE CHARGE IS BROKEN - POSSIBLE IN THE 2HDM

SUPPOSE WE LIVE IN A 2HDM, ARE WE IN DANGER?

The 2HDM potential and its minima

1. Start by writing the potential, which for the 2HDM is just a function $V(\Phi_1, \Phi_2)$

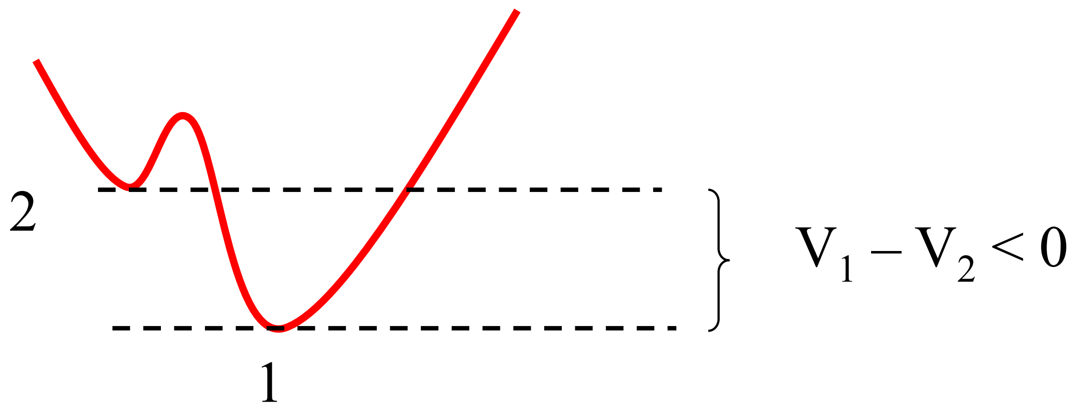
2. Find the stationary points (SP) of V

3. Classify the SP (minima, saddle points, maxima) – meaning: look at the values of the squared masses

4. You will find three types of SP – the CP-conserving (aka normal), the charge breaking and the CP breaking SP

5. You just have to write the potential at each of the SP and call it V_N, V_{CB} and V_{CP}, respectively

6. Compare the depths of the different V at each SP



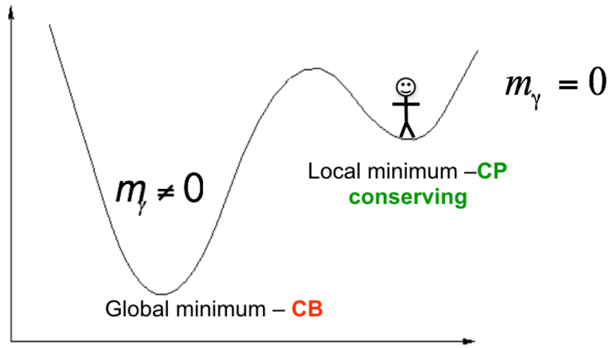
The 2HDM potential and its minima

$$V_{CB} - V_{\mathcal{N}} = \frac{m_{H^\pm}^2}{2v^2}\left[(v_2 v_1' - v_1 v_2')^2 + v_1^2 \alpha^2\right]$$

Difference of the values of the potential at the CB SP and at the N SP

If N is a minimum (note that the charged Higgs mass is calculated at the N SP)

$$V_{CB} - V_{\mathcal{N}} = \frac{m_{H^\pm}^2}{2v^2}\left[(v_2 v_1' - v_1 v_2')^2 + v_1^2 \alpha^2\right] > 0$$



We get

$$V_{\mathcal{N}} < V_{CB}$$

It can also be shown that not only the N minimum is below the CB SP, but the CB SP is a saddle point.

Valid for the most general 2HDM

A similar result holds for the simultaneous existence of a N and a CP breaking minima.

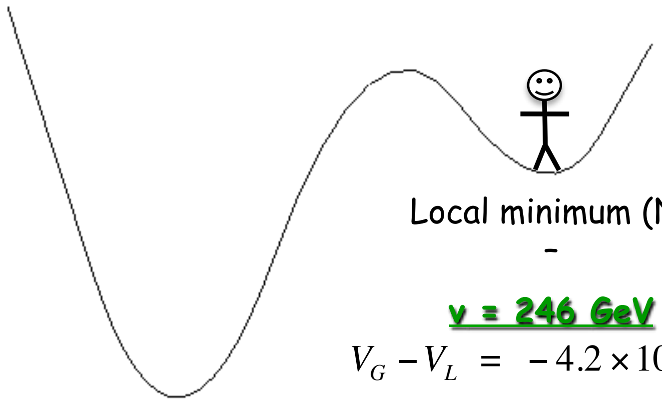
$$V_{CB} - V_{\mathcal{N}} = \frac{m_A^2}{2v^2}\left[(v_2 v_1' - v_1 v_2')^2 + v_1^2 \delta^2\right]$$

The 2HDM potential and its minima

1. 2HDM have at most two minima
2. Minima of different nature never coexist
3. Unlike Normal, CB and CP minima are uniquely determined
4. If a 2HDM has only one normal minimum, it is the absolute minimum - all other SP if they exist are saddle points
5. If a 2HDM has a CP-breaking minimum, it is the absolute minimum - all other SP if they exist are saddle points

But there is still the possibility of having two CP-conserving minima!

Two normal minima - potential with the soft breaking term



$$V_{\mathcal{N}_2} - V_{\mathcal{N}_1} = \frac{1}{2}\left\{\left(\frac{m^2_{H^\pm}}{v^2}\right)_{\mathcal{N}_2} - \left(\frac{m^2_{H^\pm}}{v^2}\right)_{\mathcal{N}_1}\right\}\left[(v_2 v_1' - v_1 v_2')^2\right]$$

Local minimum (N) - $m_W = 80.4$ GeV

v = 246 GeV

$$V_G - V_L = -4.2 \times 10^8 \text{ GeV}$$

Global minimum (N) - $m_W = 107.5$ GeV

v = 329 GeV

THE PANIC VACUUM!

and this is one that can actually occur...

However, two CP-conserving minima can coexist - we can force the potential to be in the global one by using a simple condition.

$$\mathrm{D} = m_{12}^2\left(m_{11}^2 - k^2 m_{22}^2\right)(\tan\beta - k) \quad k = \left(\frac{\lambda_1}{\lambda_2}\right)^{1/4}$$

$$D = \frac{1}{8v^8\, s_\beta^4\, c_\beta^2}\left(-a_1\,\mu^2 + b_1\right)\left(a_2\,\mu^2 - 2\,b_2\right)$$

Our vacuum is the global minimum of the potential if and only if D > 0.

The 2HDM potential and its minima

The most general potential for an NHDM is

$$V = \mu_{ij}^2 (\Phi_i^\dagger \Phi_j) + \lambda_{ijkl} (\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l)$$

From 2 to infinity

where the indices range from 1 to N and the parameters can be complex.

We have shown that a basis can be chosen such that the comparison between SP reduces to the case of 3 doublets for charge breaking and to the 2HDM case for CP breaking. So the main results are:

- In a NHDM CB minima can coexist with CP-conserving ones - the 2HDM is a very peculiar model

- In a NHDM CP minima cannot coexist with CP-conserving ones - the 2HDM result holds for an arbitrary number of doublets

The N2HDM potential and its minima

What if we start adding singlets? The most general potential for the N2HDM invariant under

$$\Phi_1 \to \Phi_1; \quad \Phi_2 \to -\Phi_2; \quad \Phi_S \to \Phi_S \qquad\qquad \Phi_1 \to \Phi_1; \quad \Phi_2 \to \Phi_2; \quad \Phi_S \to -\Phi_S$$

softly broken by the m^2_{12} term is

$$V = m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + h.c.) + \frac{m_S^2}{2}\Phi_S^2$$
$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\frac{\lambda_5}{2}\left[(\Phi_1^\dagger\Phi_2) + h.c.\right] + \frac{\lambda_6}{4}\Phi_S^4 + \frac{\lambda_7}{2}(\Phi_1^\dagger\Phi_1)\Phi_S^2 + \frac{\lambda_8}{2}(\Phi_2^\dagger\Phi_2)\Phi_S^2$$

- **The non-dark matter phase Ns, CPs, CBs** $\quad \langle \Phi_S \rangle = v_S$

  Three CP-even scalars instead of 2 as in the 2HDM

- **The dark matter phase N, CP, CB** $\quad \langle \Phi_S \rangle = 0$

  2HDM spectrum plus a dark matter candidate

The N2HDM potential and its minima

Since the most general vacuum for the N2HDM is

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{cb} \\ v_2 + i v_{cp} \end{pmatrix} \qquad \langle \Phi_S \rangle = v_S$$

with the usual definition of the charge operator.

P: How many different dark phases can this model have?

The N2HDM potential and its minima

The most general vacuum for the N2HDM is

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{cb} \\ v_2 + i v_{cp} \end{pmatrix} \qquad \langle \Phi_S \rangle = v_S$$

with the usual definition of the charge operator. Also, the potentials are all explicitly CP-conserving. Now, the possible CP-conserving and non-charge breaking minima, are

$$\langle \Phi_1 \rangle_{\mathcal{N}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \langle \Phi_2 \rangle_{\mathcal{N}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \qquad \langle \Phi_S \rangle_{\mathcal{N}} = 0$$

$$\langle \Phi_1 \rangle_{\mathcal{N}_s} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \langle \Phi_2 \rangle_{\mathcal{N}_s} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \qquad \langle \Phi_S \rangle_{\mathcal{N}_s} = v_s$$

$$\langle \Phi_1 \rangle_{\mathcal{S}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \langle \Phi_2 \rangle_{\mathcal{S}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \langle \Phi_S \rangle_{\mathcal{S}} = v_s$$

and the extra possibilities where either v_1=0 or v_2=0 which lead to (Inert-like models)

$$v_1 = 0;\ v_2 \neq 0 \quad \text{or} \quad v_1 \neq 0;\ v_2 = 0 \quad \Rightarrow \quad m_{12}^2 = 0$$

The N2HDM potential and its minima

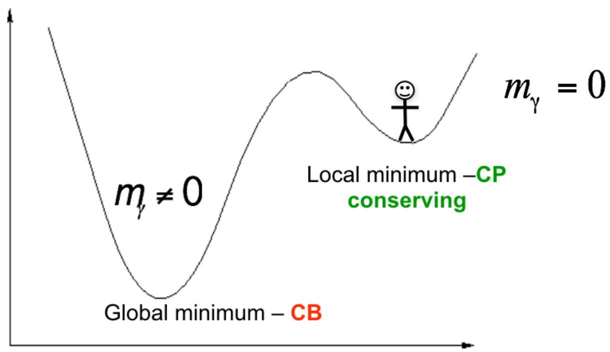
So, for this particular scenario where we compare the two "dark-matter-like" phases, one CP-conserving and the other CB, we get the exact same result as for the 2HDM

$$V_{CB} - V_{\mathcal{N}} = \frac{m_{H^\pm}^2}{4v^2}\left[(v_2 c_1 - v_1 c_3)^2 + v_1^2 c_2^2\right]$$

Difference of the values of the N2HDM potential at the CB SP and at the N SP

Again, if N is a minimum (note that the charged Higgs mass is calculated at the N SP)

$$V_{CB} - V_{\mathcal{N}} = \frac{m_{H^\pm}^2}{4v^2}\left[(v_2 c_1 - v_1 c_3)^2 + v_1^2 c_2^2\right] > 0$$



We get

$$V_{\mathcal{N}} < V_{CB}$$

It can also be shown that that not only the N minimum is below the CB SP, but the CB SP is a saddle point.

However, for the N2HDM we now have two CP-conserving SP (N and Ns) and two charge breaking SP (CB and CBs). Will this hold for all?

The N2HDM potential and its minima

$$V_{CB} - V_{\mathcal{N}} = \frac{m_{H^\pm}^2}{4v^2}\left[(v_2 c_1 - v_1 c_3)^2 + v_1^2 c_2^2\right]$$ ✅

$$V_{CBs} - V_{\mathcal{N}} = \frac{1}{4}\left\{\frac{m_{H^\pm}^2}{v^2}\left[(v_2 c_1' - v_1 c_3')^2 + v_1^2 {c_2'}^2\right] + m_D^2 {c_4'}^2\right\}$$ ✅

$$m_D^2 = m_S^2 + \frac{1}{2}(\lambda_7 v_1^2 + \lambda_8 v_2^2)$$ calculated in the N stationary point (DM mass)

$$V_{CBs} - V_{\mathcal{N}_s} = \left(\frac{m_{H^\pm}^2}{4v^2}\right)_{\mathcal{N}_s}\left[(v_2' c_1' - v_1' c_3')^2 + {v_1'}^2 {c_2'}^2\right]$$ ✅

$$V_{CB} - V_{\mathcal{N}_s} = \left(\frac{m_{H^\pm}^2}{4v^2}\right)_{\mathcal{N}_s}\left[(v_2' c_1 - v_1' c_3)^2 + {v_1'}^2 {c_2}^2\right] - \frac{1}{4}s^2 m_{s1}^2$$ ❌

$$m_{s1}^2 = m_s^2 + \lambda_7 c_1^2/2 + \lambda_8(c_2^2 + c_3^2)/2$$ calculated at the CB SP (positive if CB is a minimum)

If N is a minimum both the CB and CBs SP are above it (same as for the 2HDM).

However for the Ns minimum, a charge breaking minimum can be below the CP-conserving one!

The N2HDM potential and its minima

$$V_{CP} - V_{\mathcal{N}} = \frac{m_A^2}{4v^2}\left[(v_2c_1 - v_1c_3)^2 + v_1^2c_2^2\right]$$ ✅

$$V_{CPs} - V_{\mathcal{N}} = \frac{1}{4}\left\{\frac{m_A^2}{v^2}\left[(v_2c_1' - v_1c_3')^2 + v_1^2c_2'^2\right] + m_D^2c_4'^2\right\}$$ ✅

$m_D^2 = m_S^2 + \frac{1}{2}(\lambda_7 v_1^2 + \lambda_8 v_2^2)$ calculated in the N stationary point (DM mass)

$$V_{CPs} - V_{\mathcal{N}_s} = \left(\frac{m_A^2}{4v^2}\right)_{\mathcal{N}_s}\left[(v_2'c_1' - v_1'c_3')^2 + v_1'^2c_2'^2\right]$$ ✅

$$V_{CP} - V_{\mathcal{N}_s} = \left(\frac{m_A^2}{4v^2}\right)_{\mathcal{N}_s}\left[(v_2'c_1 - v_1'c_3)^2 + v_1'^2c_2^2\right] - \frac{1}{4}s^2m_{s1}^2$$ ❌

$m_{s1}^2 = m_s^2 + \lambda_7 c_1^2/2 + \lambda_8(c_2^2 + c_3^2)/2$ calculated at the CP SP (positive if CP is a minimum)

If N is a minimum both the CP and CPs SP are above it (same as for the 2HDM).

However for the Ns minimum, a CP-breaking minimum can be below the CP-conserving one!

The N2HDM potential and its minima

Finally the CP-conserving minima (should we keep on panicking?). Here we can have coexisting N minima, coexisting Ns minima and also N with Ns, such that

$$V_{\mathcal{N}_s} - V_{\mathcal{N}} = \frac{1}{4}\left[\left(\frac{m^2_{H^\pm}}{4v^2}\right)_{\mathcal{N}} - \left(\frac{m^2_{H^\pm}}{4v^2}\right)_{\mathcal{N}_s}\right](v_1 v_2' - v_2 v_1')^2 + \frac{1}{4} m_D^2 s^2$$

$$V_{\mathcal{N}'} - V_{\mathcal{N}} = \frac{1}{4}\left[\left(\frac{m^2_{H^\pm}}{4v^2}\right)_{\mathcal{N}} - \left(\frac{m^2_{H^\pm}}{4v^2}\right)_{\mathcal{N}'}\right](v_1 v_2' - v_2 v_1')^2$$

$$V_{\mathcal{N}'_s} - V_{\mathcal{N}_s} = \frac{1}{4}\left[\left(\frac{m^2_{H^\pm}}{4v^2}\right)_{\mathcal{N}_s} - \left(\frac{m^2_{H^\pm}}{4v^2}\right)_{\mathcal{N}_s'}\right](v_1 v_2' - v_2 v_1')^2$$

Besides the trivial minimum at the origin there is still a CP conserving possibility - the case where only the singlet acquires a VEV. This would lead to massless electroweak gauge bosons and massless fermions and would require

$$\langle \Phi_S \rangle^2 = -\frac{2 m_S^2}{\lambda_6} \;\Rightarrow\; V_S = -\frac{m_S^2}{2\lambda_6}$$

Not a single SP of this type was found in the scan.

The N2HDM potential and its minima

Extrema	$\mathcal{N}$	$\mathcal{N}s$	$\mathcal{CB}$	$\mathcal{CB}s$	$\mathcal{CP}$	$\mathcal{CP}s$	$\mathcal{S}$
$\mathcal{N}$	$\times$	$\times$	Stability	Stability	Stability	Stability	$\times$
$\mathcal{N}s$	$\times$	$\times$	$\times$	Stability	$\times$	Stability	$\times$

Stability here means absolute stability at tree-level. Are there meta-stable minima?

Scan of the N2HDM parameter space using ScannerS. We generated parameter points where the EW vacuum is of type Ns - most interesting case for vacuum stability. All parameter points fulfil the applied theoretical constraints and are compatible with the applied current experimental constraints at the 2σ level.

Theoretical constraints: tree-level unitarity, boundedness from below.

Experimental constraints: bounds from flavour physics, electroweak precision, collider physics with HiggsBounds and HiggsSignals. Branching ratios and total widths with N2HDECAY and production with SusHi. One of the CP-even, neutral Higgs masses is fixed to 125.09 GeV.

We do not impose absolute stability of the EW vacuum as a theoretical constraint since we want to study the vacuum structure in detail and take into account that metastable regions of the parameter space are allowed.

The N2HDM potential and its minima

In imposing vacuum stability constraints we distinguish the following cases:

- parameter points where the EW vacuum is the only vacuum,
- absolutely stable parameter points where secondary minima exist but are never deeper,
- long-lived parameter points where secondary vacua are deeper but never dangerous,
- short-lived parameter points that have dangerous secondary minima.

The value of the scalar potential at each of these stationary points is compared to the depth of the EW vacuum.

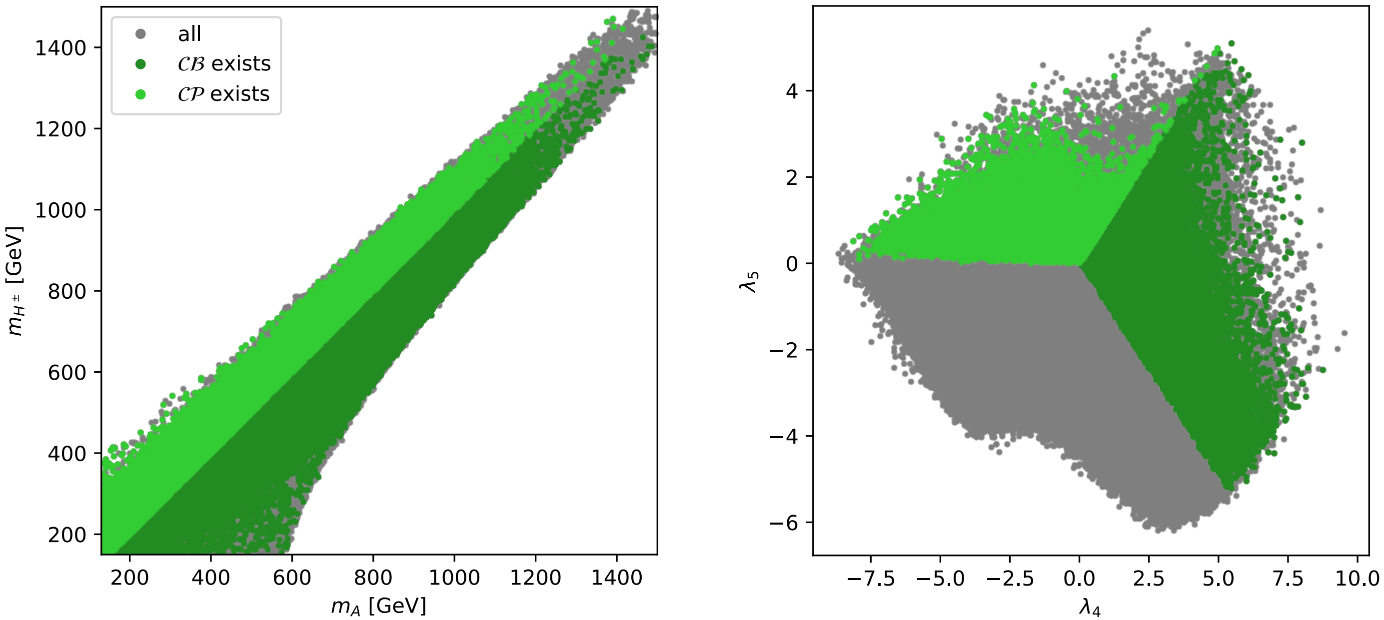
If there is no stationary point deeper than the EW vacuum we consider the EW vacuum at this parameter point as absolutely stable.

If stationary points deeper than the EW vacuum exist we calculate the tunnelling time to each of these deeper extrema.

	$\mathcal{N}s\,'$	$\mathcal{N}$	$\mathcal{CB}$	$\mathcal{CP}$
exists	0.05%	23.3%	4.49%	2.80%
deep	0.0015%	20.9%	4.11%	2.55%
dangerous	0%	6.89%	1.12%	0.678%

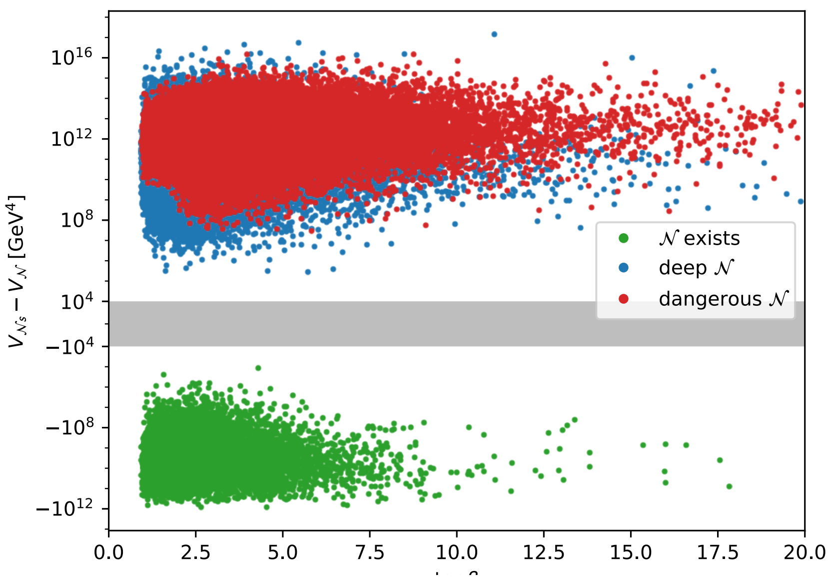
The N2HDM potential and its minima

Coexistence of minima



The distribution of secondary charge and CP breaking minima. Left: plane of the CP-odd Higgs mass m_A and charged Higgs mass $m_{H^\pm}$. Right: λ_4 vs. λ_5. In grey points fulfilling all theoretical and experimental constraints; in dark green (light green) a secondary minimum of type CB (CP) exists.

The N2HDM potential and its minima

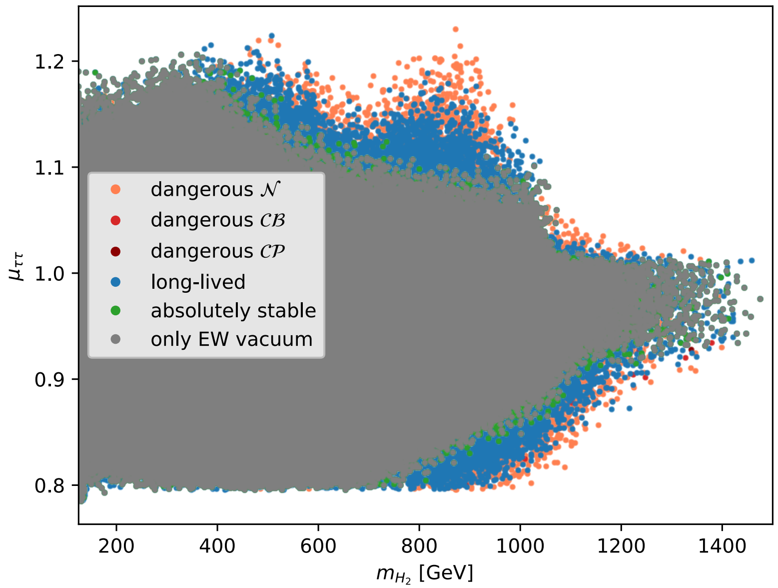


The difference in the value of the scalar potential between the EW Ns vacuum and a secondary N minimum as a function of tan β at the EW vacuum. Only parameter points where a secondary N vacuum exists are shown.

Green: parameter points have a secondary N minimum but tunnelling from the EW vacuum is not possible.

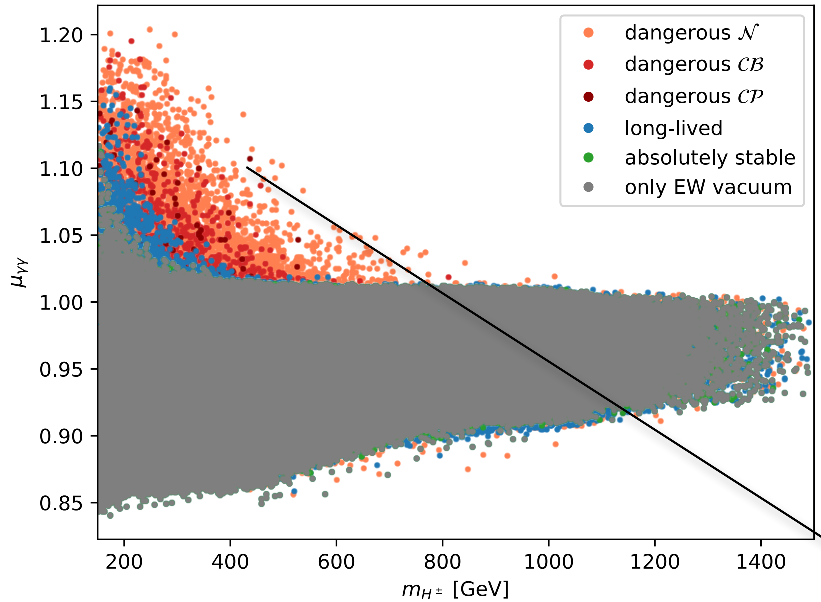
Blue: points tunnelling is possible but slow

Red: short-lived tunnelling to the N minimum.

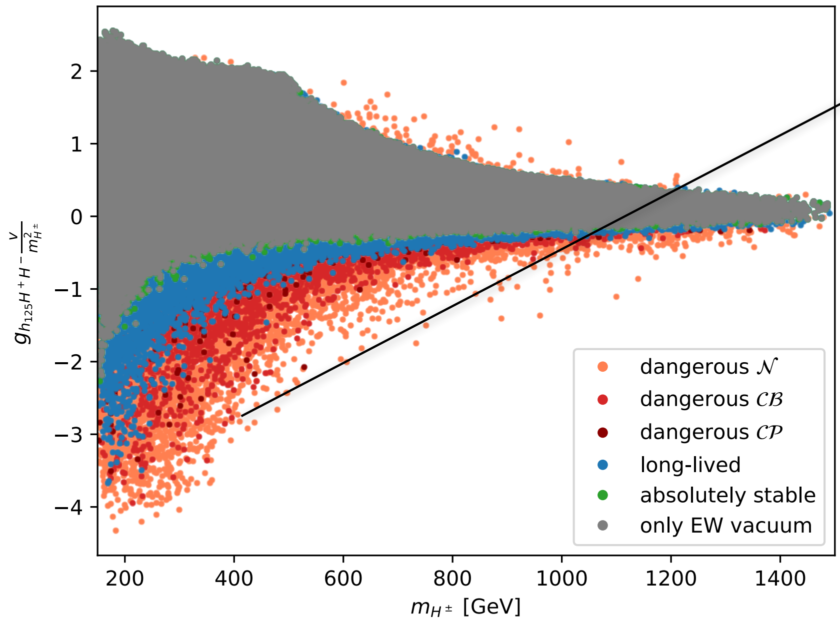


The signal strength $\mu_{\tau\tau}$ of $h_{125} \to \tau\tau$ as a function of the second lightest neutral scalar mass m_{H_2}. The parameter points without any secondary minima (grey) are plotted on top, followed by the absolutely stable (green), and long-lived (blue) parameter points.

Below these, the points with dangerous secondary minima are shown in different shades of red denoting the type of dangerous minimum present (N – light red, CB – red, CP – dark red).



The signal strength $\mu_{\gamma\gamma}$ of $h_{125} \rightarrow \gamma\gamma$ as a function of the charged Higgs mass. The parameter points without any secondary minima (grey) are plotted on top, followed by the absolutely stable (green), and long-lived (blue) parameter points. Below these, the points with dangerous secondary minima are shown in different shades of red denoting the type of dangerous minimum present (N – light red, CB – red, CP – dark red).



Excluded regions in the coupling to the charged Higgs and in the rate. And this was the only observable where a clear difference was found.

$$\mu_{\gamma\gamma} = \frac{\sigma(pp \to h_{125})\mathrm{BR}(h_{125} \to \gamma\gamma)}{\sigma(pp \to h_{\mathrm{SM}})\mathrm{BR}(h_{\mathrm{SM}} \to \gamma\gamma)}$$

The normalised coupling $g_{h_{125}H^+H^-}$ as a function of the charged Higgs mass.

The one-loop zero temperature potential - aka Coleman-Weinberg potential

The 1-loop effective potential

We would like to have the potential at all orders in perturbation theory. With the full potential the minima can again shift and new stationary points can appear. We start (again) with the simple example of a scalar theory with a Z_2 symmetry.

$$\mathscr{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - V_0(\phi) \qquad V_0(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4$$

The 1-loop corrections to the tree-level potential are computed as the sum of all 1PI diagrams with a single loop and zero external momentum

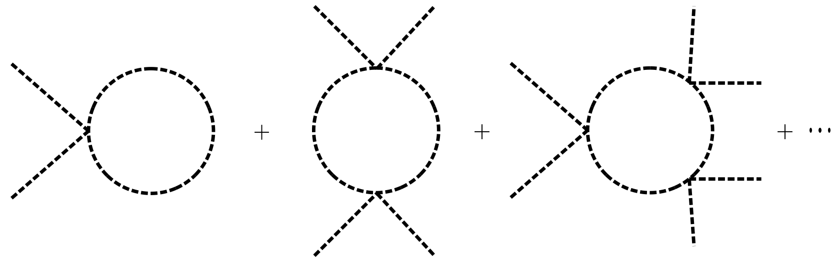


Figure 1: 1PI diagrams contributing to the one-loop effective potential of (22).

The diagram of order n has n propagators and 2n external legs. The n propagators contribute with a factor $i^n(p^2 - m^2 + i\epsilon)^{-n}$, the external lines contribute a factor ϕ_c^{2n} and each vertex contributes with $-i\lambda/2$. There is also a global $1/(2n)$. Finally there is an integration over the loop momentum and an extra factor i.

The 1-loop effective potential

The effective potential is written as

$$V_{eff}(\phi_c) = V_0(\phi_c) + V_1(\phi_c)$$

with

$$V_1(\phi_c) = i \sum_{n=1}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{1}{2n} \left(\frac{\lambda \phi_c^2/2}{p^2 - m^2 + i\epsilon} \right)^n = \frac{-i}{2} \int \frac{d^4p}{(2\pi)^4} \log \left(1 - \frac{\lambda \phi_c^2/2}{p^2 - m^2 + i\epsilon} \right)$$

After a Wick rotation $p^0 = ip_E^0, p_E = (-ip^0, \vec{p}), p^2 = (p^0)^2 - \vec{p}^2 = -p_E^2$ we can write

$$V_1(\phi_c) = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \log \left(1 + \frac{\lambda \phi_c^2/2}{p_E^2 + m^2} \right)$$

and using the shifted masses

$$m^2(\phi_c) = m^2 + \frac{1}{2}\lambda \phi_c^2 = \frac{d^2 V_0(\phi_c)}{d\phi_c^2}$$

we can finally write, dropping the index E and neglecting the field independent term (more later)

$$V_1(\phi_c) = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \log \left(p^2 + m^2(\phi_c) \right)$$

The 1-loop effective potential

The result can be generalised for N scalar complex fields

$$\mathscr{L} = \frac{1}{2}\partial^\mu \phi^a \partial_\mu \phi_a^\dagger - V_0(\phi)(\phi^a, \phi_a^\dagger)$$

The one-loop contribution to the effective potential is

$$V_1 = \frac{1}{2} Tr \int \frac{d^4 p}{(2\pi)^4} \log\left(p^2 + M_s^2(\phi^a, \phi_b^\dagger\right) \qquad (M_s^2)_b^a = V_b^a = \frac{\partial^2 V}{\partial \phi_a^\dagger \partial \phi^b}$$

with $Tr(M_s^2) = 2V_a^a$, where the factor 2 comes from the fact that each complex field contains two degrees of freedom. Similarly $Tr(1) = 2N$.

The one-loop contribution to the effective potential in the case of Nf fermions and Ng gauge bosons is

$$V_1 = -2\lambda \frac{1}{2} Tr \int \frac{d^4 p}{(2\pi)^4} \log\left[p^2 + M_f^2(\phi_c)\right] \qquad \text{Fermions}$$

$$V_1 = Tr(\Delta) \frac{1}{2} Tr \int \frac{d^4 p}{(2\pi)^4} \log\left[p^2 + (M_{gb})^2(\phi_c)\right] \qquad \text{Gauge bosons}$$

$$Tr(\Delta) = 3$$

The 1-loop effective potential

The results for the one-loop potentials all have UV infinities. So we need to renormalise them. Let us do this with a simple example of a massless scalar theory

$$\mathcal{L} = \frac{1}{2}(1+\delta Z)(\partial_\mu \phi)^2 - \frac{1}{2}\delta m^2 \phi^2 - \frac{\lambda + \delta\lambda}{4!}\phi^4$$

Before we start let us write again the general renormalisation conditions. There are two free parameters in the theory and therefore we have two renormalisation conditions (where we will hide the infinities)

$$m_R^2 = -\Gamma^{(2)}(p=0) = \left.\frac{d^2 V}{d\phi_c^2}\right|_{\phi_c=0}$$ Condition for the mass

$$\lambda_R = -\Gamma^{(4)}(p=0) = \left.\frac{d^4 V}{d\phi_c^4}\right|_{\phi_c=0}$$ Condition for the coupling

$Z(0) = 1$ Condition for the field - which we don't really need

Now we have to put the potential in a usable form.

The 1-loop effective potential

Let us start by using an energy cut-off Λ and use the following result from integrating in the angular variables

$$\int d^n p f(\rho) = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2})} \int f(\rho)\rho^{n/2-1}\, d\rho$$

where $\rho = |p|^2$. Show that

$$V_1(\phi_c) = \frac{1}{32\pi^2}\int_0^{\Lambda^2} \rho \log[\rho + m^2(\phi_c)] d\rho.$$

Neglecting field independent terms and terms that vanish in the limit $\Lambda \to \infty$ show that

$$V_1(\phi_c) = \frac{1}{32\pi^2} m^2(\phi_c)\Lambda^2 + \frac{1}{64\pi^2} m^4(\phi_c)\left[\log \frac{m^2(\phi_c)}{\Lambda^2} - \frac{1}{2}\right]$$

Meaning the full one-loop potential is

$$V = \frac{1}{2}\delta m^2 \phi_c^2 + \frac{\lambda + \delta\lambda}{4!}\phi_c^4 + \frac{\lambda \phi_c^2}{64\pi^2}\Lambda^2 + \frac{\lambda^2\phi_c^4}{256\pi^2}\left(\log \frac{\lambda\phi_c^2}{2\Lambda^2} - \frac{1}{2}\right)$$

The 1-loop effective potential

The renormalisation conditions for this particular model are chosen such that

$$\left.\frac{d^2V}{d\phi_c^2}\right|_{\phi_c=0} = 0$$ Condition for the mass

$$\lambda = \left.\frac{d^4V}{d\phi_c^4}\right|_{\phi_c=\mu}$$ Condition for the coupling

Show that the counterterms are

$$\delta m^2 = -\frac{\lambda}{32\pi^2}\Lambda^2$$

$$\delta\lambda = -\frac{11\lambda^2}{32\pi^2} - \frac{3\lambda^2}{32\pi^2}\log\frac{\lambda\mu^2}{2\Lambda^2}$$

Which leads to the potential

$$V_{\rm eff} = \frac{\lambda}{4!}\phi_c^4 + \frac{\lambda^2\phi_c^4}{256\pi^2}\log\left(\frac{\phi_c^2}{\mu^2} - \frac{25}{6}\right)$$

The 1-loop effective potential

Using dimensional regularisation the result is

$$V_{\text{eff}} = \frac{\lambda}{4!}\phi_c^4 + \frac{\lambda^2\phi_c^4}{256\pi^2}\log\left(\frac{\lambda\phi_c^2}{2\mu^2} - \frac{3}{2}\right)$$

$$V_1(\phi_c) = -\lambda\frac{1}{32\pi^2}M_f^4(\phi_c)\left\{\log\frac{M_f^2(\phi_c)}{\mu^2} - \frac{3}{2}\right\}$$

Fermions

$$V_1(\phi_c) = 3\frac{1}{64\pi^2}M_{gb}^4(\phi_c)\left\{\log\frac{M_{gb}^2(\phi_c)}{\mu^2} - \frac{5}{6}\right\}$$

Gauge bosons

The results for the SM are

$$V(\phi_c) = V_0(\phi_c) + \frac{1}{64\pi^2}\sum_{i=W,Z,h,\chi,t} n_i m_i^4(\phi_c)\left[\log\frac{m_i^2(\phi_c)}{\mu^2} - C_i\right]$$

$$C_W = C_Z = \frac{5}{6}$$
$$C_h = C_\chi = C_t = \frac{3}{2}$$

$$n_W = 6,\ n_Z = 3,\ n_h = 1,\ n_\chi = 3,\ n_t = -12$$

The temperature dependent potential

The T dependent potential

We start with the partition function and will first take care of the boson case

$$Z(T) = \mathrm{Tr}\left[e^{-\beta \hat{H}}\right]$$

$$\begin{aligned} F &= -T \ln Z\,, && \text{Free energy} \\ S &= -\frac{\partial F}{\partial T}\,, && \text{Entropy} \\ E &= -\frac{\partial \ln Z}{\partial \beta}\,, && \text{Energy} \end{aligned}$$

Starting with the harmonic oscillator

$$\hat{H}\,|n\rangle = \omega\left(n+\frac{1}{2}\right)|n\rangle$$

$$\begin{aligned} Z_{\rm bho}(T,\omega) &= \sum_{n=0}^{\infty} \exp\left[-\beta\omega\left(n+\frac{1}{2}\right)\right] = \frac{e^{-\beta\omega/2}}{1-e^{-\beta\omega}}, && \text{Partition function} \\ F_{\rm bho}(T,\omega) &= \frac{1}{2}\omega + T\,\ln\left(1-e^{-\beta\omega}\right), && \text{Free energy} \end{aligned}$$

And for the free Klein-Gordon field

$$\hat{\phi}(\vec{x},t) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{2\omega_{\vec{k}}}\left(\hat{a}_{\vec{k}}e^{-i\vec{k}\cdot\vec{x}} + \hat{a}^\dagger_{\vec{k}}e^{i\vec{k}\cdot\vec{x}}\right)$$

the free energy density is

$$f_B = \frac{F_{\rm B}}{\mathcal{V}} = V_{0,B} + T\int\frac{\mathrm{d}^3k}{(2\pi)^3}\ln\left(1-e^{-\beta\omega_{\vec{k}}}\right)$$

$V_{0,B}$ is the energy density of the zero-temperature ground state, which is divergent. The same divergence is encountered in QFT at zero temperature. The standard renormalisation convention takes the zero-temperature ground state free energy to be zero.

The T dependent potential

Due to the integration over all momenta, f_B can only depend on T and m, where m only appears as m/T. From dimensional analysis we infer that the free energy density hence takes the form

$$f_B(T,m) = T^4 J_B\left(\frac{m}{T}\right)$$

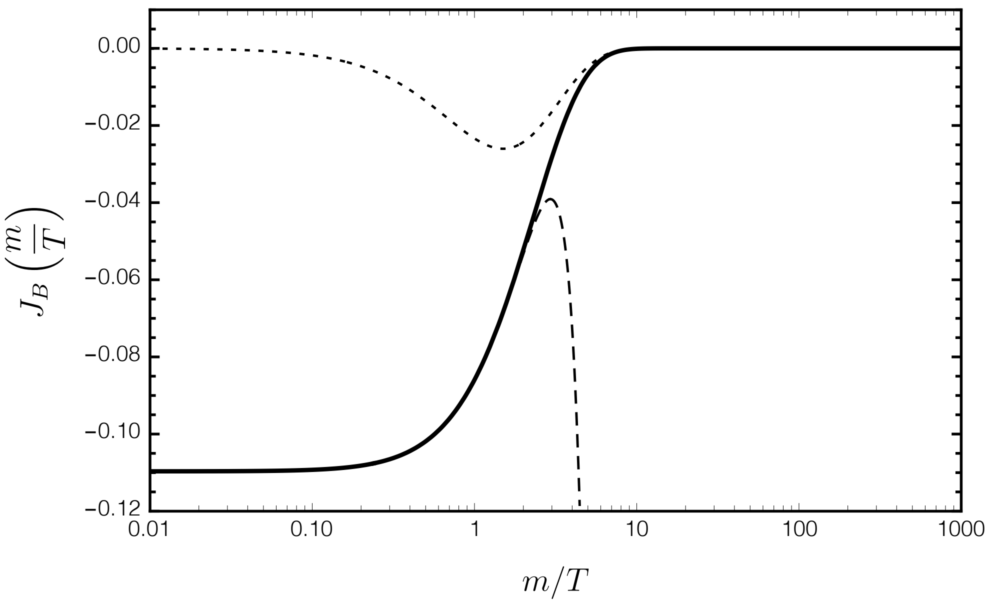


Figure 1: This figure shows the dimensionless function J_B that is proportional to the free energy of bosons as defined in Eq. (2.19), as a function of mass-to-temperature ratio (thick line). Also the expansions for large T (dashed), Eq. (2.21) and small T (dotted), Eq. (2.20) are shown. The large-T expansion is performed up to order four in m/T, being a good approximation up to $m/T \sim 1.1$.

Low temperature

$$J_B\left(\frac{m}{T}\right) = -\left(\frac{m}{2\pi T}\right)^{\frac{3}{2}} e^{-m/T}\left(1+\mathcal{O}\left(\frac{T}{m}\right)\right)$$

High temperature

$$\begin{aligned} J_B\left(\frac{m}{T}\right) &= -\frac{\pi^2}{90}+\frac{1}{24}\left(\frac{m}{T}\right)^2-\frac{1}{12\pi}\left(\left(\frac{m}{T}\right)^2\right)^{\frac{3}{2}} \\ &\quad -\frac{1}{2(4\pi)^2}\left(\frac{m}{T}\right)^4\left[\ln\left(\frac{1}{4\pi}\frac{m}{T}e^{\gamma_E}\right)-\frac{3}{4}\right] \\ &\quad +\mathcal{O}\left(\left(\frac{m}{T}\right)^6\right). \end{aligned}$$

And for fermions

$$f_F = T^4 J_F\left(\frac{m}{T}\right)$$

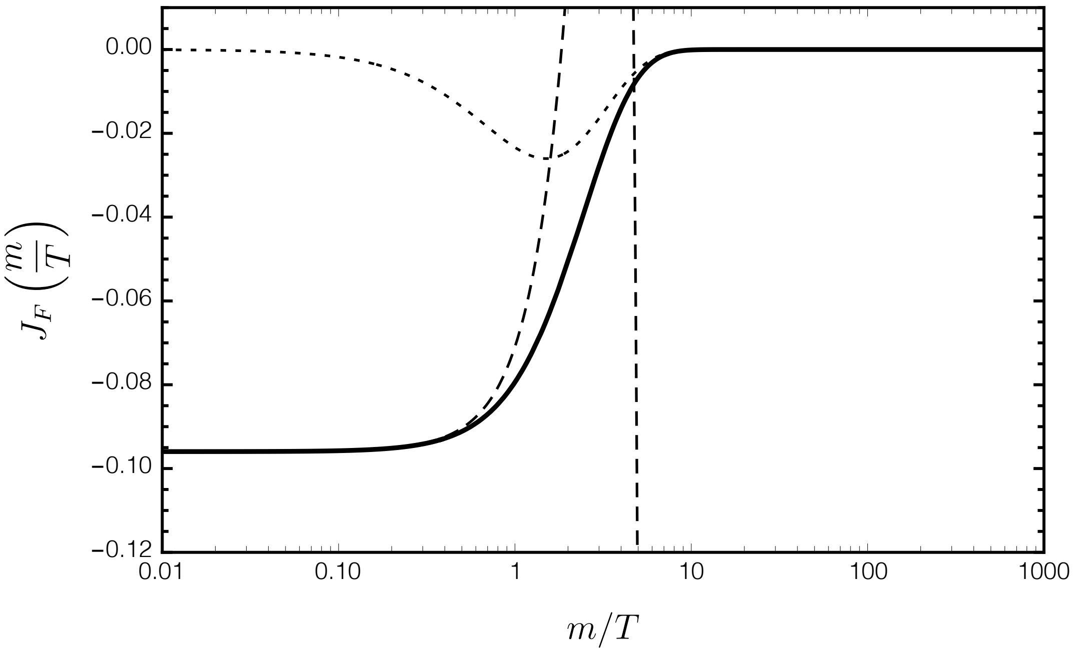


Figure 2: This figure shows the dimensionless function J_F that is proportional to the free energy of fermions as defined in Eq. (2.27) as a function of mass-to-temperature ratio (thick line). Additionally, the expansion for large T (dashed), Eq. (2.28) and small T (dotted) , in analogy to Eq. (2.20) are shown. Note that in the small T limit, both the fermionic and the bosonic expansions agree. The large T expansion is performed up to order four in m/T, working well up to $m/T \sim 0.5$, hence being sligthly worse than the bosonic high-T expansion, depicted in Fig. 1 .

The T dependent potential

The complete expression for the temperature dependent potential of the SM is

$$\Delta V^{(1)}(\phi_c, T) = \frac{T^4}{2\pi^2}\left[\sum_{i=W,Z} n_i J_B[m_i^2(\phi_c)/T^2] + n_t J_F[m_t^2(\phi_c)/T^2]\right]$$

Which mean that we can write the complete potential as

$$V(\phi_c, T) = D(T^2 - T_o^2)\phi_c^2 - ET\phi_c^3 + \frac{\lambda(T)}{4}\phi_c^4$$

with

$$D = \frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v^2}$$

$$E = \frac{2m_W^3 + m_Z^3}{4\pi v^3}$$

$$T_o^2 = \frac{m_h^2 - 8Bv^2}{4D}$$

$$B = \frac{3}{64\pi^2 v^4}\left(2m_W^4 + m_Z^4 - 4m_t^4\right)$$

$$\lambda(T) = \lambda - \frac{3}{16\pi^2 v^4}\left(2m_W^4 \log\frac{m_W^2}{A_B T^2} + m_Z^4 \log\frac{m_Z^2}{A_B T^2} - 4m_t^4 \log\frac{m_t^2}{A_F T^2}\right)$$

$$\log A_B = \log a_b - 3/2 \text{ and } \log A_F = \log a_F - 3/2,$$

$$a_f = \pi^2 \exp(3/2 - 2\gamma_E) \ (\log a_f = 2.6351)$$

$$a_b = 16\pi^2 \exp(3/2 - 2\gamma_E) \ (\log a_b = 5.4076)$$

Daisy resummations

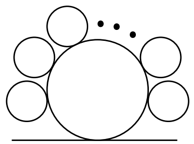
As for T = 0, the mass term receives higher-order corrections as (scalar theory again)

$$\bar{M}^2 = \bar{m}^2 + \Sigma_\varphi(\varphi; T),$$

where Σ_φ is the temperature dependent self-energy. The leading contribution is

$$\bar{M}^2 = \bar{m}^2 + \frac{\lambda}{2} I(\bar{m}^2) = \bar{m}^2 + \frac{\lambda}{2}\left[\frac{T^2}{12} - \frac{T\bar{m}}{4\pi} + \cdots\right]$$

The most important corrections come from Daisy diagrams and are of the order of



$$\text{(daisy)} \sim \frac{\lambda^2 T^3}{\bar{m}} \left(\frac{\lambda T^2}{\bar{m}^2}\right)^{n-2}.$$

Therefore, for a scalar theory not only we need to have $\lambda < 1$ but also $\lambda T^2/m^2 < 1$ in order that the perturbative expansion makes sense (the diagrams are summed).

The final result for the scalar mass correction is (introduced by redefining the masses - Debye masses)

$$\bar{M}^2 = \bar{m}^2 + \frac{\lambda}{2} I\left(\bar{m}^2 + \frac{\lambda T^2}{24}\right)$$

Phase transitions

1st and 2nd order phase transitions

Let us first consider the potential

$$V(\phi, T) = D(T^2 - T_o^2)\phi^2 + \frac{\lambda(T)}{4}\phi^4$$

The potential stationary points are

$$\phi(T) = 0 \qquad\qquad \phi(T) = \sqrt{\frac{2D(T_o^2 - T^2)}{\lambda(T)}}$$

And the second derivative is

$$m^2(\phi, T) = 3\lambda\phi^2 + 2D(T^2 - T_o^2)$$

At $T > T_0$ - only the solution $\phi = 0$ exists and $m^2(0,T) > 0$ which means the solution is a minimum.

At $T = T_0$ - both solutions collapse at $\phi = 0$ and $m^2(0,T) = 0$.

At $T < T_0$ - both solutions exist. Show that there is one minimum and one maximum.

When the broken phase is formed, the origin (symmetric phase) becomes a maximum. The phase transition may be achieved by a thermal fluctuation for a field located at the origin. There is no energy barrier to overcome - <u>second order phase transition</u>.

1st and 2nd order phase transitions

Let us consider now the complete potential (note that all the terms are present)

$$V(\phi, T) = D(T^2 - T_o^2)\phi^2 - ET\phi^3 + \frac{\lambda(T)}{4}\phi^4$$

The potential stationary points are

$$\phi(T) \;=\; 0 \qquad \phi_M(T) = \frac{3ET}{2\lambda(T)} - \frac{1}{2\lambda(T)}\sqrt{9E^2T^2 - 8\lambda(T)D(T^2 - T_o^2)} \quad \phi_m(T) = \frac{3ET}{2\lambda(T)} + \frac{1}{2\lambda(T)}\sqrt{9E^2T^2 - 8\lambda(T)D(T^2 - T_o^2)}$$

There is a temperature below $T = T_1$ for which only the solution $\phi = 0$ exists. Show that this temperature in given by

$$T_1^2 = \frac{8\lambda(T_1)DT_o^2}{8\lambda(T_1)D - 9E^2}$$

There is a temperature (which we call the critical temperature) $T = T_c$ for which we have two minima at the same height. Show that this is indeed the case and show that:

$$T_c^2 = \frac{\lambda(T_c)DT_o^2}{\lambda(T_c)D - E^2} \qquad \phi_M(T_c) = \frac{ET_c}{\lambda(T_c)} \qquad \phi_m(T_c) = \frac{2ET_c}{\lambda(T_c)}$$

At $T = T_0$ for the barrier disappears and the origin becomes a maximum. Show that

$$\phi_M(T_o) = 0 \qquad \phi_m(T_o) = \frac{3ET_o}{\lambda(T_o)}$$

1st and 2nd order phase transitions

The phase transition starts at T = T_c by tunnelling. The chronology of the phase transition is

At $T > T_1$ - only the solution $\phi = 0$ exists.

At $T = T_1$ - a local inflection point appears for at $\phi = 3ET_1/(2\lambda(T_1))$.

At $T < T_1 \quad (T > T_c)$ - both solutions exist and there is a minimum and a maximum away from the origin.

At $T = T_c$ - we have two minima one at the origin and another one at a value $\phi = 2ET_c/\lambda(T_c)$. There is also a maximum.

At $T = T_0$ - the barrier disappears and and the origin becomes a maximum.

In this case a barrier is formed and the phase transition proceeds via tunnelling. There is an energy barrier to overcome - first order phase transition.

Phase transitions

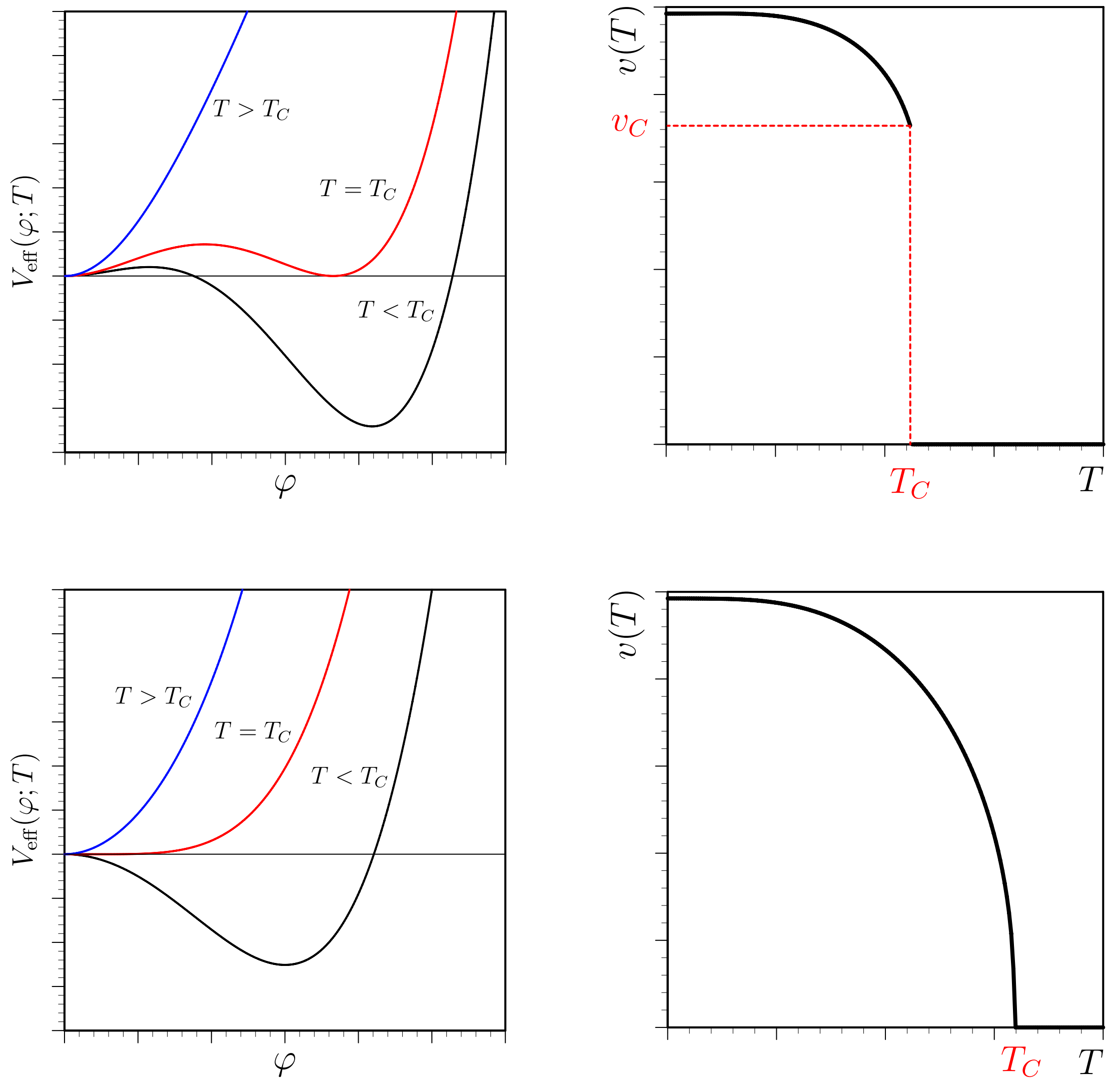
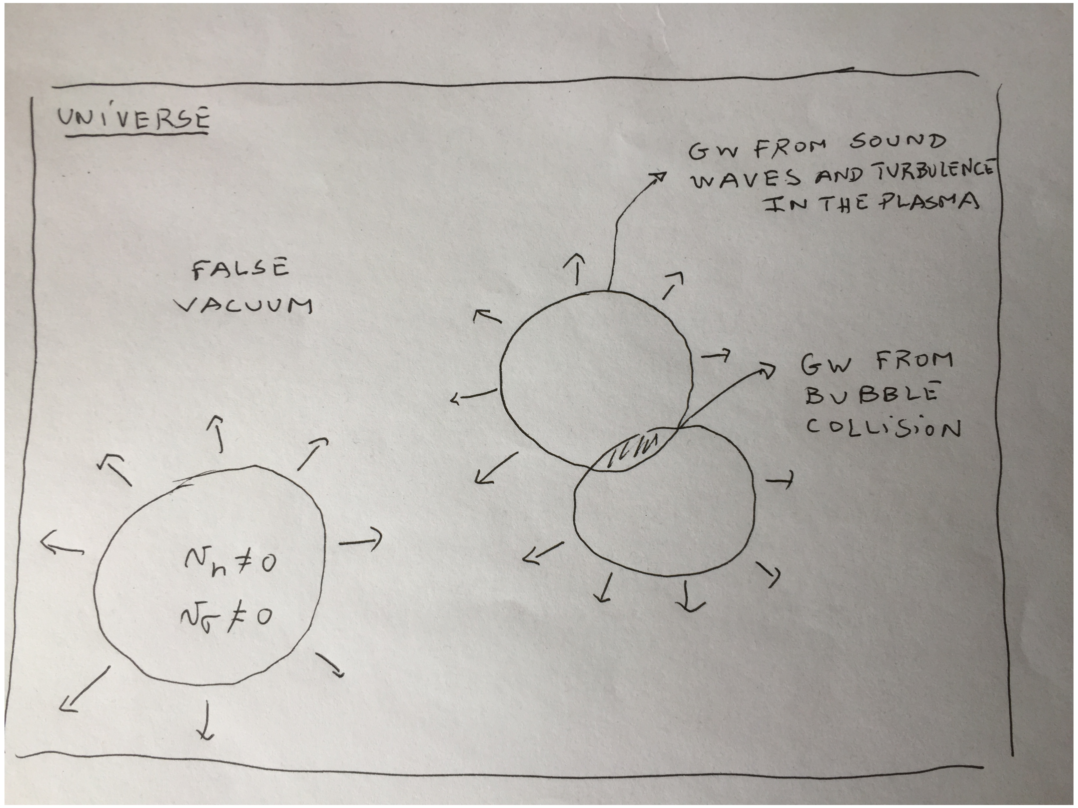


Figure 1. Two types of phase transitions. (Upper) Case of the first-order phase transition; shapes of the effective potential at $T > T_C$, $T = T_C$ and $T < T_C$ [left panel] and the temperature evolution of the VEV of scalar [right panel]. (Lower) Counterparts in the case of the second-order phase transition.

Phase transitions

Sakharov conditions for baryogenesis ask for a SFOPT (EW) in the course of the thermal evolution of the Universe to prevent dilution of the generated baryon asymmetry.

In a SFOPT the universe changes from a symmetric (metastable high energy) phase to a broken phase through the nucleation of bubbles of the broken phase.



For a vacuum transition the bubbles nucleate through quantum tunnelling, and during expansion the bubble wall travels at close to the speed of light.

The bubble collisions are an extremely violent process that may give rise to gravitational waves.

GW may be detectable by future space based gravitational wave observatories such as LISA, and if so would be able to tell us about the conditions in the early universe.

Thermal tunnelling

Transition from the false to the true vacuum proceeds via thermal tunnelling at finite temperature. We describe it as the formation of bubbles of the broken phase in the sea of the symmetric phase. After that the bubbles spread throughout the universe converting false vacuum into true one. It happens after the critical temperature (meaning at a lower temperature).
The rate of bubble nucleation (more later) is given by

$$\Gamma(T) = A(T) e^{-\hat{S}_3/T}$$

Where A(T) is roughly proportional to T^4 and $\hat{S}_3/T$ is the O(3) symmetric Euclidian action,

$$\hat{S}_3(\hat{\phi}, T) = 4\pi \int_0^\infty \mathrm{d}r\, r^2 \left\{ \frac{1}{2} \left(\frac{\mathrm{d}\hat{\phi}}{\mathrm{d}r} \right)^2 + V_{\mathrm{eff}}(\hat{\phi}, T) \right\} ,$$

With $\hat{\phi}$ being the field VEVs that follow the classical path

$$\frac{d^2\hat{\phi}}{dr^2} + \frac{2}{r}\frac{d\hat{\phi}}{dr} = \frac{V_{eff}}{d\hat{\phi}}$$

with the boundary conditions

$$\hat{\phi}(r)|_{r\to\infty} = 0; \qquad \frac{d\hat{\phi}}{dr}\Big|_{r\to 0} = 0$$

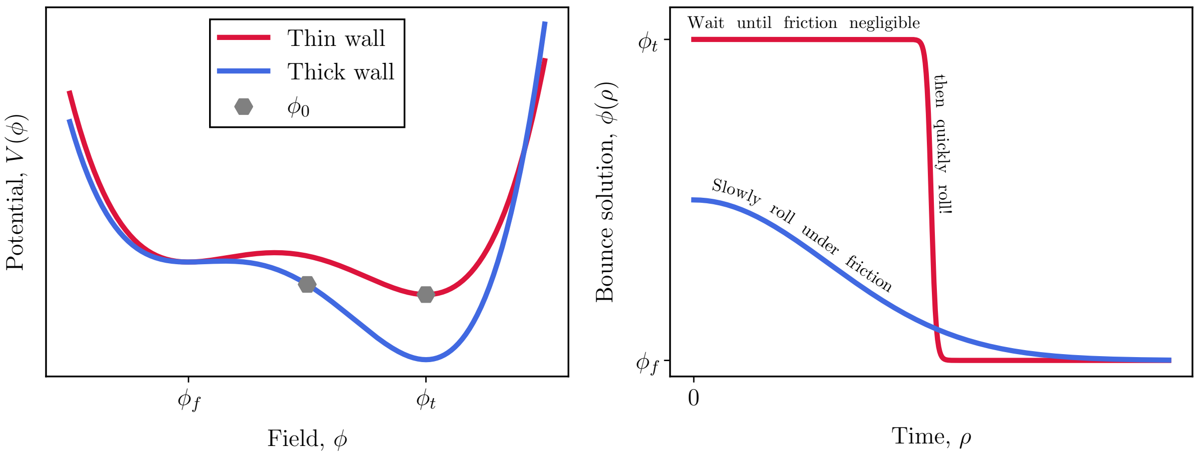
Thermal tunnelling

The rate of bubble nucleation depends on the 3-action below. We take $\phi = 0$ outside the bubble

$$\hat{S}_3(\hat{\phi}, T) = 4\pi \int_0^R \mathrm{d}r\, r^2 \left\{ \frac{1}{2} \left(\frac{\mathrm{d}\hat{\phi}}{\mathrm{d}r} \right)^2 + V_{\mathrm{eff}}(\hat{\phi}, T) \right\} ,$$

There are two contributions: a surface term FS , coming from the derivative term, and a volume term FV , coming from the second term. They scale like

$$\hat{S}_3(\hat{\phi}, T) \sim 2\pi R^2 \left(\frac{\delta\hat{\phi}}{\delta R} \right)^2 \delta R + \frac{4\pi R^3 < V >}{3}$$



For the scalar potential an analytic formula has been obtained without assuming the thin wall approximation. It is given by,

$$\frac{S_3}{T} = \frac{13.72}{E^2} \left[D \left(1 - \frac{T_o^2}{T^2} \right) \right]^{3/2} f \left[\frac{\lambda(T) D}{E^2} \left(1 - \frac{T_o^2}{T^2} \right) \right] \qquad f(x) = 1 + \frac{x}{4} \left[1 + \frac{2.4}{1-x} + \frac{0.26}{(1-x)^2} \right]$$

Bubble nucleation

Now we have the critical radius of a bubble large enough to grow after formation. If the phase transition is completed or not, depends on the ratio of the rate of production of bubbles of true vacuum, over the expansion rate of the universe.

The phase transition will start at some temperature T_n (the nucleation temperature) by bubble nucleation. The probability of bubble formation per unit time and per unit volume is given by

$$\frac{\Gamma}{\nu} \sim A(T)e^{-S_3/T} \qquad A(T) \approx \omega T^4$$

where w is taken to be of order 1.

A homogeneous and isotropic (flat) universe is described by a Robertson-Walker metric where a(t) is the scale factor of the universe. The universe expansion is governed by the equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_{P\ell}^2}\rho$$

where M_{Pl} is the Planck mass, and ρ is the energy density. For temperatures T ~ 10^2 GeV the universe is radiation dominated, and its energy density is given by

$$\rho = \frac{\pi^2}{30}g(T)T^4$$

where where g(T) is the effective degrees of freedom.

Bubble nucleation

The onset of nucleation happens at a temperature T_n such that the probability for a single bubble to be nucleated within one horizon volume is

$$\int_{T_t}^{\infty} \frac{dT}{T} \left(\frac{2\zeta M_{\rm Pl}}{T} \right)^4 \exp\{-S_3(T)/T\} = \mathcal{O}(1) \qquad \zeta = \frac{1}{4\pi}\sqrt{\frac{45}{\pi g}} \sim 3 \times 10^{-2}$$

And this implies numerically that

$$B(T_t) \sim 137 + \log \frac{10^2 E^2}{\lambda D} + 4 \log \frac{100\ GeV}{T_t} \qquad \breve{B}(T) = S_3(T)/T$$

Giving the nucleation temperature.

Relevant temperatures

Critical Temperature - T_c - The potential has two degenerate minima and, consequently, the transition from the false vacuum to the true vacuum begins via quantum tunnelling.

Nucleation Temperature - T_n - The temperature at which the tunnelling decay rate matches the Hubble rate.

$$\frac{\Gamma(T_n)}{H^4(T_n)} = 1$$

Percolation Temperature - T_* - Temperature at which at least 34% of the false vacuum has tunnelled into the true vacuum or, equivalently, the probability of finding a point still in the false vacuum is 70%.

$$P(T) = e^{-I(T)}, \qquad I(T) = \frac{4\pi v_b^3}{3} \int_T^{T_c} \frac{\Gamma(T')dT'}{T'^4 H(T')} \left(\int_T^{T'} \frac{d\tilde{T}}{H(\tilde{T})} \right)^3 ,$$

To find the percolation temperature one needs to solve $I(T_*) = 0.34$. Note that

$$T_* < T_n < T_c$$

Strong 1st order phase transitions in the SM

We already discussed the T-dependent potential for the SM written as

$$V(\phi, T) = D(T^2 - T_o^2)\phi^2 - ET\phi^3 + \frac{\lambda(T)}{4}\phi^4 \qquad E = \frac{2m_W^3 + m_Z^3}{4\pi v^3}$$

We also know that the minimum is at

$$\phi_m(T_c) = \frac{2ET_c}{\lambda(T_c)}$$

The relation between the Higgs mass, the VEV, and the quartic coupling is $m_h^2 = 2\lambda v^2$. For the 1st order phase transition to be strong we need that

$$\frac{\phi(T_c)}{T_c} \gtrsim 1.3$$

Now write a bound on the Higgs mass and comment.

Phase transitions in the scalar extension

- Scenario 1 - SM + Complex singlet - only the doublet acquires a VEV, $\varphi_\sigma(T = 0) = 0$. At finite temperatures though, the real part may fluctuate around a non-zero $\varphi_\sigma(T)$. Two possible dark matter (DM) candidates. One was always a DM particle since the beginning of the Universe while the other, for certain none zero temperatures, featured a temperature dependent mixing with the neutral component from the doublet, vanishing as $T \to 0$. Interaction between the dark sector and the SM only via the quartic portal coupling.

- Scenario 2 - SM + Complex singlet - both the doublet and the real component of the gauge singlet acquire VEVs at $T = 0$, that is, $\varphi_{h,\sigma}(T = 0) \equiv v_{h,\sigma}$. One of the CP-even scalar states is identified with the SM-like Higgs boson with a mass of 125 GeV. The second scalar, that mixes with the SM-like Higgs boson, can be either heavier or lighter than the 125 GeV Higgs boson candidate in this case. The soft breaking term in the potential explicitly breaks $U(1)_L \to Z_2$ providing a pseudo-Goldstone mass to the imaginary part of the field σ_I.

Phase transitions in the scalar extension

- Scenario 1 and 2 - the tree-level Higgs potential is the same for all 3 scenarios

$$\mathscr{V}_0(\Phi,\sigma) = \mu_\Phi^2 \Phi^\dagger\Phi + \lambda_\Phi(\Phi^\dagger\Phi)^2 + \mu_\sigma^2\sigma^\dagger\sigma + \lambda_\sigma(\sigma^\dagger\sigma)^2 + \lambda_{\Phi\sigma}\Phi^\dagger\Phi\sigma^\dagger\sigma + \left(\frac{1}{2}\mu_b^2\sigma^2 + \mathrm{h.c.}\right),$$

with

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} G + iG' \\ \phi_h + h + i\eta \end{pmatrix}, \qquad \sigma = \frac{1}{\sqrt{2}}(\phi_\sigma + \sigma_R + i\sigma_I),$$

In scenario 1 at zero temperature the mass spectrum is

$$m_h^2 = 2\lambda_\Phi v_h^2, \quad m_{D1}^2 = \mu_\sigma^2 + \mu_b^2 + \frac{\lambda_{\Phi\sigma}v_h^2}{2}, \quad m_{D2}^2 = \mu_\sigma^2 - \mu_b^2 + \frac{\lambda_{\Phi\sigma}v_h^2}{2}, \quad m_{D2}^2 - m_{D1}^2 = -2\mu_b^2,$$

and the 6 free parameters are chosen as the 125 GeV Higgs mass, the doublet VEV, the 2 DM particle masses and the two quartic couplings from the dark sector.

In scenario 2 there is mixing between the two Higgs

$$m_{h_1,h_2}^2 = \lambda_\Phi v_h^2 + \lambda_\sigma v_\sigma^2 \mp \frac{\lambda_\sigma v_\sigma^2 - \lambda_\Phi v_h^2}{\cos 2\theta} \qquad m_D^2 \equiv m_{\sigma_I}^2 = -2\mu_b^2, \qquad \mu_b^2 < 0.$$

and the 6 free parameters are the 125 GeV Higgs, the doublet VEV, the mass of the new scalar, the mass of the DM particle, the singlet VEV and the mixing angle.

Phase transitions in the scalar extension

The 1-loop finite temperature potential is given by

$$V_{eff}(T) = V_0 + V_{CW}^{(1)} + V_{ct} + \Delta V$$

The counterterm potential is given by

$$V_{\rm ct} = \delta\mu_\Phi^2 \Phi^\dagger\Phi + \delta\lambda_\Phi \left(\Phi^\dagger\Phi\right)^2 + \delta\mu_\sigma^2 \sigma^\dagger\sigma + \delta\lambda_\sigma \left(\sigma^\dagger\sigma\right)^2 + \delta\lambda_{\Phi\sigma}\Phi^\dagger\Phi\sigma^\dagger\sigma + \left(\frac{1}{2}\delta\mu_b^2\sigma^2 + \text{ h.c. }\right)$$

The CT potential is introduced so that by applying the following renormalization conditions, the tadpole equations and mass terms are unchanged at 1-loop

$$\left\langle \frac{\partial V_{\rm ct}}{\partial h_i} \right\rangle = -\left\langle \frac{\partial V_{\rm CW}^{(1)}}{\partial h_i} \right\rangle, \qquad \left\langle \frac{\partial^2 V_{\rm ct}}{\partial h_i \partial h_j} \right\rangle = -\left\langle \frac{\partial^2 V_{\rm CW}^{(1)}}{\partial h_i \partial h_j} \right\rangle .$$

The temperature corrections are

$$\Delta V(T) = \frac{T^4}{2\pi^2}\left\{ \sum_b n_b J_B\left[\frac{m_i^2(\phi_\alpha)}{T^2}\right] - \sum_f n_f J_F\left[\frac{m_i^2(\phi_\alpha)}{T^2}\right]\right\}, \qquad J_{B/F}(y^2) = \int_0^\infty dx\, x^2 \log\left(1 \mp \exp[-\sqrt{x^2+y^2}]\right)$$

where n_b (n_f) are the bosonic (fermionic) d.o.f. for each particle b (f) in the summation, and m_i are the field dependent masses. $J_{B/J}$ functions are the bosonic (fermionic) thermal integrals.

Phase transitions in the scalar extension

The T^2 terms in the thermal expansion are modified by the inclusion of an all-order resummation procedure (so-called daisy diagrams). In practice, this is done by a correction to the tree-level potential mass terms given by

$$\mu_\alpha^2(T) = \mu_\alpha^2 + c_\alpha T^2$$

In scenarios 1 and 2 this means the addition of

$$c_h = \frac{3}{16}g^2 + \frac{1}{16}g'^2 + \frac{1}{2}\lambda_\Phi + \frac{1}{12}\lambda_{\Phi\sigma} + \frac{1}{4}(y_t^2 + y_b^2 + y_c^2 + y_s^2 + y_u^2 + y_d^2) + \frac{1}{12}(y_\tau^2 + y_\mu^2 + y_e^2),$$

$$c_\sigma = \frac{1}{3}\lambda_\sigma + \frac{1}{6}\lambda_{\Phi\sigma},$$

Plus in scenario 3

$$c_\sigma \to c_\sigma + \frac{1}{24}\sum_{i=1}^{6} Y_{\sigma_i}^2$$

and plus the following additions for the longitudinal modes of the gauge bosons

$$m_{W_L}^2(\phi_h; T) = m_W^2(\phi_h) + \frac{11}{6}g^2 T^2,$$

$$m_{Z_L,A_L}^2(\phi_h; T) = \frac{1}{2}m_Z^2(\phi_h) + \frac{11}{12}(g^2 + g'^2)T^2 \pm \mathcal{D}, \qquad \mathcal{D}^2 = \left(\frac{1}{2}m_Z^2(\phi_h) + \frac{11}{12}(g^2 + g'^2)T^2\right)^2 - \frac{11}{12}g^2 g'^2 T^2\left(\phi_h^2 + \frac{11}{3}T^2\right)$$

Phase transitions in the scalar extension

The strength of the phase transitions is related to the Latent Heat and given by

$$\alpha = \frac{1}{\rho_\gamma}\left[V_i - V_f - \frac{T_*}{4}\left(\frac{\partial V_i}{\partial T} - \frac{\partial V_f}{\partial T}\right)\right],$$

where ρ_γ is the radiation density outside the bubble.

The inverse time scale of the transitions if given by (time of phase transition is β^{-1})

$$\frac{\beta}{H} = T_* \frac{\partial}{\partial T}\left(\frac{\hat{S}_3}{T}\right)\Bigg|_{T_*}$$

The order parameter of the phase transition is given by

$$\frac{\Delta v_\phi}{T_*} = \frac{|v_\phi^f - v_\phi^i|}{T_*}, \qquad \phi = h, \sigma$$

The spectrum of the GW is given by

$$h^2\Omega_{\rm GW} = h^2\Omega_{\rm GW}^{\rm peak}\left(\frac{4}{7}\right)^{-\frac{7}{2}}\left(\frac{f}{f_{\rm peak}}\right)^3\left[1 + \frac{3}{4}\left(\frac{f}{f_{\rm peak}}\right)\right]^{-\frac{7}{2}}$$

where $f_{\rm peak}$ is proportional to the inverse of the mean bubble separation.

Gravitational waves

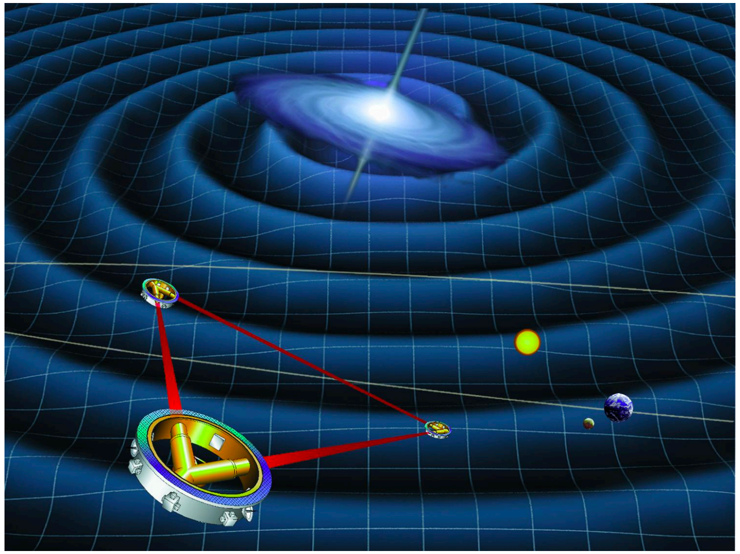
Gravitational waves and LISA

GWs are decoupled from the rest of matter and radiation components in the universe. GWs propagate freely in the early universe, immediately after they are generated.

They carry information about the processes that produced them. We can access the state of the Universe at epochs and energy scales unreachable by any other means.

Complementarity with the LHC or future particle colliders.

From NASA LISA site



“LISA consists of three spacecraft that are separated by millions of miles. These spacecraft relay laser beams back and forth between the different spacecraft and the signals are combined to search for gravitational wave signatures that come from distortions of spacetime.”

“A bit like the objects moving on the surface of a pond produce ripples and waves, massive objects moving in space distort the fabric of spacetime and produce gravitational waves. Some of these gravitational wave events will cause the three LISA spacecraft to shift slightly with respect to each other, as they "ride the gravitational waves", to produce a characteristic pattern in the combined laser beam signal that depends on the location and physical properties of the source.”

Relevant quantities

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where f_{peak} is proportional to the inverse of the mean bubble separation.

THE END